Анализ критериев самоторможения

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Self-braking Criteria Analysis

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Приведен анализ понятия самоторможения и известных аналитических и геометрических критериев этого явления. Показана некорректность использования коэффициента полезного действия в качестве критерия. Предложено исследовать явление самоторможения с помощью параметра торможения звена, равного взятому с обратным знаком отношению элементарных работ внутренних сил сопротивления и движущих сил. Критически проанализирована известная классификация самотормозящихся механизмов, а также предложен принципиально иной взгляд на такие механизмы, согласно которому они не образуют отдельного класса, поскольку явление самоторможения присуще всем механизмам при определенных геометрических параметрах и условиях движения. Описаны высокоэффективные конструкции самотормозящихся механизмов. На примере цилиндрической зубчатой передачи показано использование вероятностного метода определения запаса самоторможения.

Ключевые слова: аналитические и геометрические критерии, самоторможение, критерий самоторможения, запас самоторможения, вероятностный метод.

The purpose of the article is self-braking criteria validity and practical usage analysis.

The term «braking» originates from Greek тормос, which literally means the hole to insert a nail to prevent wheel rotation. Currently term «a break» relates to a unit which decreases velocity or provides stoppage of the machine [1]. Break blocks the mechanism at necessary mode of operation, for example, prevents mechanism motion due to the applied weight.

Blocking may be obtained without any special device just by providing excessive friction in any of kinematic pairs preventing further motion of the mechanism. This phenomenon was named self-breaking. It is well known and widely used in worm and screw gears. Different definitions and phenomenon criteria are met so far in the literature.

In N.I. Levitsky's manual [2] self-breaking is described as a case when friction force in a prismatic kinematic pair doesn’t allow «relative motion of a link in the desired direction independently of the magnitude of the resultant moving force». Self-breaking condition of a kinematic pair is expressed by inequality

\[ a_i \leq 0_{ij}, \] (1)
where $a_i$ is angle between driving force, applied to $i$-link, and the normal to the guide; $\theta_{ij}$ is angle between reaction acting on link $i$ from link $j$ and the normal to the guide.

Inequality (1) means that self-breaking takes place when line of action of the driving force lies inside friction angle. The deviation angle of this line relatively to the normal is the factor of self-breaking.

In manual [3] self-breaking phenomenon is illustrated in double wedge mechanism. Conditions for possibility of forward and reverse run are given in Table 1. Reverse run is impossible if angle of the wedge is $\gamma < 2\theta$ [3]. It is also mentioned that reverse run is possible if to make the force applied to the input driving by changing its direction to the opposite. If driving forces are applied to both — input and output links — it is wrong to classify this situation as reverse run mode.

The mode of motion when both input and output links are driving is a distinctive feature of self-breaking mechanisms. V.L. Veits names this mode as «break release» operation [4]. In this case break release coefficient $\mu_{i+1,i}$ serves as a factor of loss. It is expressed as the ratio of powers at the input $i$ and output $i + 1$ links and differs from efficiency coefficient by opposite sign: $\mu_{i+1,i} = \eta_{i+1,i}$.

Typically self-breaking is used to prevent reverse run of a mechanism. Though, as V.L. Veits noted, self-breaking mechanisms may exist at any moment direction, but their practical usage is questionable [4]. A.K. Musatov is even more strict: «In this case the mechanism is no operable and has no application» [3].

A.I. Turpayev names braking action at forward run as seizure of improperly designed mechanism. He names a mechanism as self-breaking «if it may start motion by the driving (input) link, any attempt to start mechanism by driven (output) link causes braking action, the force flow acts at the frame (housing)>> [5].

Table 1

<table>
<thead>
<tr>
<th>Wedge angle $\gamma$</th>
<th>Forward run</th>
<th>Reverse run</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma &lt; 2\theta$</td>
<td>Possible</td>
<td>Impossible</td>
</tr>
<tr>
<td>$2\theta &lt; \gamma &lt; \frac{\pi}{2} - 2\theta$</td>
<td>Possible</td>
<td>Possible</td>
</tr>
<tr>
<td>$\gamma &gt; \frac{\pi}{2} - 2\theta$</td>
<td>Impossible</td>
<td>Possible</td>
</tr>
</tbody>
</table>

Profesor A.F. Krainev defines self-breaking as «a condition at which due to friction force relative motion of the links cannot start independently of the magnitude of the driving force» [1]. This definition has two distinctive features. First, term «self-breaking» is applied to define the condition instead of the phenomenon. Second, any mechanism blocked from motion by some external friction mechanism may correspond to the definition. Relative motion of the mechanism links cannot start due to friction, but friction forces do not appear in kinematic pairs of the mechanism. They are the result of interaction with the external friction mechanism.

Some preliminary conclusions on self-breaking features of one degree of freedom mechanisms may be done based on the given references:

• self-breaking means the fact that mechanism cannot start motion in one of the directions even if there is no resisting friction;

• self-breaking emerges due to high friction in kinematic pairs;

• when self-breaking takes place the condition $0 \leq \eta < 1$ isn’t obtained, formally calculated efficiency is negative;

• self-breaking may be overcome by mutual correlated action of input and output links when both of them become driving links; this mode is named break release, mode of accord forces;

• self-breaking may be useful at reverse run, at forward run it is usually unacceptable.

Comparison of given definitions shows substantial differences of the authors’ approaches to the phenomenon. The differences are connected with the views on self-breaking phenomenon considered as:

• case, condition or effect; taking place in mechanisms or kinematic pairs;

• depending on the direction of driving force or links geometry;

• whether its criteria are angle of driving force vector relatively to the normal and the efficiency of the corresponding motion mode.

Differences of the definitions given in the references presented show that the notion of self-breaking isn’t established yet due to insufficient phenomenon research. Self-breaking notion includes two phenomena: slowness of motion and retention at the state of rest. They may take place due to internal and external forces acting on the mechanism. These phenomena should be analyzed separately for complete description of self-breaking effect.
Let’s name a mechanism as self-braking if slowdown is due to internal, relatively to the mechanism, forces acting independently of the magnitude of driving forces. Let’s differ self-braking of motion, considered is slowdown, and self-braking of standstill considered as retaining in this condition. In other words, self-breaking is mechanism slowdown exceptionally due to friction in its kinematic pairs independently of the driving forces.

This definition specifies the entity of self-braking phenomenon which takes place due to the internal friction forces in the mechanism. At standstill condition possible work of internal friction forces equals the work of driving forces (independently of their magnitude). At self-braking mode work of internal friction forces exceeds the work of driving forces that leads to mechanisms slowdown up to its stoppage.

Both self-braking definitions and its criteria presented in the references differ. Two groups of criteria may be allocated. For the first group, the factor describing self-braking is the position of the total force relatively to the pressure angle in the prismatic pair, or cone of friction in the revolute pair. These criteria may be named geometrical. Self-braking in the second group is defined by the value of efficiency coefficient of the corresponding mode of motion, typically reverse run. These criteria may be named analytical. Formal status of the analytical criteria is more applicable for calculation, mainly performed by a computer, but it lacks physical meaning. In the book [3] it is mentioned that for a mechanism in the state of self-breaking efficiency coefficient is of no physical sense as the mechanism is motionless and no work is done.

For a long time it was considered that forward run efficiency of self-breaking mechanisms is less than 50 %. Unfortunately this statement is still met in the manuals, and there are attempts to prove it [6]. In fact, in 1935 V.V. Dobrovolsky proved possibility, on principle, of self-breaking with the forward run efficiency greater 50 % . He also mentioned the nature of the mistake: at the reverse run work of harmful resistance forces was assumed equal or less then the work of the same forces at the forward run. This statement isn’t true for all mechanisms [7]. The author considered the negative value of the of efficiency coefficient \( \eta < 0 \) as the indicator of self-braking. This result is of no physical meaning [7].

V.V. Dobrovolsky warned against another mistake that may contain danger. If \( \eta < 0.5 \), it doesn’t mean automatically that the machine is self-braking. This warning is still timely. In 52 years after professor V.V. Dobrovolsky published his article [7] one may find in «Vestnik Machinostroeniy» (Machine Building Newsletter) the following [8]: «… in self-braking mechanisms \( \eta < 0.5 \); if self-breaking is undesirable and the mechanism to be used as reversible the necessary condition is \( \eta > 0.5 \) ». It is also stated referring to V.I. Panjukhin [9] that similar conclusions are made for other self-breaking mechanisms, though the article mentioned deals with self-breaking gear with high efficiency coefficient.

In 1956 A.P. Metral and I. Le Ber developed graphical method to specify efficiency of higher kinematic pairs [10]. The method may be applied to specify self-breaking condition of planar mechanisms. The method practically is applied to cam mechanisms and the possibility of its application to gearings with parallel axes is considered.

Self-breaking gear with parallel axes was proposed by A. Roano (Switzerland) [11] in 1958. Self-breaking condition was defined by the position of engagement total force which is the geometry sum of normal component and engagement sliding friction force. Thus, the author used geometrical self-breaking criterion.

From the comparison of geometrical self-breaking criterion, based on the position of driving force action line relatively to the angle or cone of friction, and analytical, based on the efficiency value, if follows that the advantage of the first is visualization, and of the second — convenience of usage, especially for computerized analysis.

Worm gear and screw-and-nut gear were the first self-breaking gear mechanisms. They found wide application in different machines and instruments due to combined functions of motion transmission and automatic braking when the engine is off.

However worm gear has substantial shortcomings: direct run low efficiency, it needs antifriction lubrication, as a rule has low load bearing capability. Its application is limited: «Worm gear of load-lifting machines can’t replace the brake» [12, article 142]. Article 135 of the document is less strict: «One of the brakes of manually actuated lifting mechanisms may be replaced by self-breaking gear» [12].

Thus, the main goals of further development of self-breaking mechanisms are substantial increase of forward run efficiency and evaluation of self-breaking reliability.

Twin-worm gear proposed in 1961 by I.B. Popper (Israel) [13] was one of the first self-
braking mechanisms with high efficiency. It provides two options of self-braking mode depending on helix angle.

Research provided by N.S. Munster [14] on method [10] implementation to obtain efficiency of gearing with parallel axes proved principal feasibility of making this gear self-braking provided forward run high efficiency. Also it was found that the two options of self-braking mode are feasible, they are similar to those of twin-worm gear.

In I.D. Howel’s (USA) gear [15] two worms with different helix angles are engaged, their axes are parallel and the engagement is beyond the pitch point. Parallel axes design eliminates typical for twin-worm poor adaptability to manufacture and excludes axial load in case of herringbone worms. The author also mentions that the two self-braking options are possible.

A.I. Turpayev proposed to subdivide existing diversity of self-braking mechanisms into three groups based on «structural and design concepts differences, efficiency range» [5]. The first group form mechanisms with permanent (elementary) structure, they are listed in the items 1–3 of his classification. Items 4–6 correspond to mechanisms with efficiency of forward run \( \eta > 0.5 \). In this case wedge mechanisms appeared simultaneously in two classification categories as their efficiency is either \( \eta > 0.5 \) or \( \eta < 0.5 \) depending of the wedge sharpening. Gear drive with parallel axis (item 6) depending on geometry characteristics may also have \( \eta > 0.5 \) or \( \eta < 0.5 \). The second group consists of mechanisms with variable structure — complex (or composite). It includes clutches, complex screw mechanisms and lifting-and-transport machines brakes (item 12). The third group (items 13–15) is two-stage mechanisms «formed of two independent mechanisms, one of which is typically ball-screw (or roll-screw) mechanism» [5]. In item 14 automatic brake is the second independent mechanism.

Inner contradiction of the mentioned classification reflects the fact that self-braking mechanism which performs driving and braking functions contains features of both. Reference of the same mechanism to different classification groups indicates that self-braking mechanisms do not form separate class of devices. It is more correct to consider conditions at which a particular mechanism becomes self-braking one.

Analytical expressions to define self-braking condition for spur involute gearing taking into account friction at bearing support and engagement rolling friction were obtained by T.G. Iskhakov in 1969 [16]. In the same year S. Botther and G. Zirig undertook a similar study of spur gearing considering friction at bearing support [17]. They come to the conclusion that spur gearing with fixed axes and traditional geometry can’t be self-braking.

A chapter in V.L. Veits’s book [4] is dedicated to dynamics of machine assembly. It contains self-braking and brake release notions, formulas expressing losses for a set of gears. Thus, for twin-worm loss characteristics are defined by:

\[
\eta_{i,i+1} = \frac{\sin \gamma_i \sin (\gamma_{i+1} + \theta^*)}{\sin \gamma_{i+1} \sin (\gamma_i + \theta^*)} \quad (2)
\]

\[
\eta_{ii,i+1} = \frac{\sin \gamma_i \sin (\gamma_i - \theta^*)}{\sin \gamma_i \sin (\gamma_{i+1} - \theta^*)} \quad (3)
\]

where \( \theta^* = \arctan \left( f \sqrt{1 + \tan^2 \alpha_x \cos^2 \gamma} \right) \) — reduced angle of friction.

It follows from expression (3), that the condition of reverse run self-braking is

\[ k_1 = \theta^* / \gamma_i \geq 1 \]

where «\( k_1 > 1 \) may be considered as self-braking assurance factor» [4].

From (3) B.L. Veits comes to a conclusion: «Seizure at brake release mode excluding any motion at any external moment (the so-called second order self-braking) takes place upon the condition

\[ k_2 = \theta^* / \gamma_{i+1} \geq 1 \]

where \( k_2 > 1 \) — self-braking assurance coefficient» [4].

Paying attention to similarity of expressions (2) and (3), instead of three loss characteristics used let’s introduce the following one: braking parameter of \( i \)-link standstill \( \tau_i \) as opposite to the ratio of elementary works of internal resistance \( \delta A_r^- \) and \( \delta A_r^+ \) of driving forces:

\[ \tau_i = - (\delta A_r^- / \delta A_r^+) \quad (4) \]

Let’s obtain the range of this factor. As elementary works in the numerator and denominator of ratio (4) are of different signs, the ratio is always negative. Thus, parameter \( \tau_i \) is always positive. Equality of this parameter to zero is impossible as resistance to motion in kinematic pairs always exists and the numerator of (4) is never zero. At the same time resistance forces elementary work at standstill condition can’t exceed the work of driving forces, thus numerator of this expression can’t be greater than the denominator. Thus, standstill
braking parameter may vary within the limits $0 < \tau_i^0 \leq 1$. While calculating resistance force work, formal value of upper limit of standstill braking parameter, characterizing its assurance $t_i = \tau_i^0 - 1$, may be obtained substituting real value of force friction by its maximal value expressed in accordance with Amonton’s law. The greater standstill braking parameter (exceeding 1), the greater is the assurance value.

Thus, for a given state of mechanism the link providing braking is the one for which standstill braking parameter complies with $\tau_i^0 \geq 1$ independently of its position and driving force magnitude. Existence of braking link in a mechanism makes the whole mechanism self-braking for given mode of operation.

In accordance to standstill braking parameter let’s introduce braking parameter of $i$-link motion as opposite to the ratio of internal resistance forces $A_i^-$ and driving forces works $A_i^+$:

$$\tau_i = -\left(\frac{A_i^-}{A_i^+}\right).$$

This parameter differs from similar expression (4) by the following: works at the final displacement are being calculated instead of elementary works, the interval of integration should be rather great; while calculating friction forces work standstill friction coefficients are being replaced by motion friction coefficients.

Deterministic approach to important initial parameter — sliding friction coefficient — is typical for the known methods of self-braking gear analysis. It means that a gear drive designed in this way provides workability at forward run and braking mode at reverse run for given interval range of friction coefficient. Meanwhile this coefficient is a random value, thus all dependencies containing it are probabilistic.

Based on probability method let’s evaluate self-braking capability of cylindrical gearing consisting of pinion and wheel. For this purpose let’s find the value of helix angle $\beta_1$ of the pinion, which provides self-braking for a given probability value $P$. Let’s introduce supplementary function $B$ [9]:

$$B(f; \beta_b) = \sin \beta_b \frac{1}{f_0^2} + \frac{1}{\cos^2 \beta_b},$$

where $f$ is sliding friction coefficient of engagement; $\beta_b$ is helix angle at base circumference.

Let’s consider function $B$ (5) as a function of a random value $f$. Let’s assume that random value is in accordance with normal law of distribution, it has expectancy $M(f) = f_0$ and mean-square deviation $\sigma(f)$. To find the necessary angle let’s expand function $B$ in a Taylor series in accordance with D.N. Reshetov’s method [18] leaving the first two items:

$$B(f; \beta_b) = \sin \beta_b \frac{1}{f_0^2} + \frac{1}{\cos^2 \beta_b} - \sin \beta_b \frac{1}{f_0^2} (f - f_0).$$

Then angle $\beta_p$ corresponding to given probability is

$$\beta_p = \arctan [M(B) - u_p \sigma(B)],$$

where $M(B)$ is mathematical expectation of $B$ function,

$$M(B) = \sin \beta_b \frac{1}{f_0^2} + \frac{1}{\cos^2 \beta_b},$$

where $u_p$ is normal distribution quantile; $\sigma(B)$ is mean quadratic deviation of $B$ function,

$$\sigma(B) = \sin \beta_b \frac{\max f - \min f}{6 f_0^2} \frac{1}{\cos^2 \beta_b}.$$ 

For geometry analysis of cylindrical self-braking gearing pinion helix angle at reverse run is assumed greater then angle $\beta_p$ by $\Delta \beta$:

$$\Delta \beta = \beta_{\beta_1} - \beta_p.$$ 

If $\Delta \beta$ increases, braking reliability also increases due to some angle allowance.

In a similar way the expression for breaking allowance $t$ may be made linear. Considering only sliding friction it takes the form of

$$t(f; \alpha_{\beta_1}; \beta_b) = f_0 \frac{\tan \alpha_{\beta_1}}{\cos \beta_b} + (f - f_0) \frac{\tan \alpha_{\beta_1}}{\cos \beta_b},$$

where $\alpha_{\beta_1}$ is profile angle at the end section on the circumference of arbitrary radius $r_{\beta_1}$.

Then braking allowance is defined by

$$t = \frac{\tan \alpha_{\beta_1}}{\cos \beta_b} \left(f_0 + u_p \frac{\max f - \min f}{6}\right).$$

Let’s illustrate the dependencies by numerical example. Let $\min f = 0.076$; $\max f = 0.124$; $f_0 = 0.1$. The task is to evaluate braking margin assuming probability $P = 0.99$. 


For standard wheel profile angle $\alpha_{ny} = 20^\circ$ at normal cross-section helix angle in accordance with self-braking condition [9] must be less than $82^\circ50'$.

Taking $\beta_2 = 82^\circ$, $\beta_b = 68^\circ31'$. Based on self-braking condition $\beta = 186^\circ26'$. Let’s assume $\beta_1 = 86^\circ30'$. In accordance with (7) and (8) expectation and mean square deviation of $B$ function are: $M(B) = 9.646$; $\sigma(B) = 0.718$.

Probability $P = 0.99$ corresponds to $u_P = -2.326$ [18], then in accordance with (6) helix angle $\beta_P = 84^\circ57'$, that provides self-braking for given probability. Difference between this angle and the assumed value of $\beta_1$ is obtained from (9): $\Delta \beta = 1^\circ33'$.

Angle $\alpha_{by} = 81^\circ04'$ corresponds to the accepted value $\beta_{1y}$. Braking allowance corresponding to given probability is obtained from (10): $t = 1.413$. Similar calculation results for different probabilities are given in Table 2, corresponding diagrams in Figure. From the diagrams it is visible that if necessary probability increases, $\beta_P$ increases. It also leads to decrease of difference $\Delta \beta$ and braking allowance $t$.

### Table 2

<table>
<thead>
<tr>
<th>$P$</th>
<th>$u_P$ [18]</th>
<th>$\beta_P$</th>
<th>$\Delta \beta$</th>
<th>$t$</th>
</tr>
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<tbody>
<tr>
<td>0.5000</td>
<td>0.000</td>
<td>83°48'</td>
<td>2°25'</td>
<td>1.736</td>
</tr>
<tr>
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<td>2°19'</td>
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<td>2°12'</td>
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<tr>
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<td>85°22'</td>
<td>1°08'</td>
<td>1.219</td>
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</table>

### CONCLUSIONS

1. Self-braking mechanisms form no separate class of mechanism; self-braking phenomenon may take place in any mechanism provided specific characteristics of mechanism and kinematic pairs operating conditions.

2. Self-braking criteria are subdivided to geometrical and analytical. The first are obvious, the second are applicable for computation.

3. Efficiency coefficient can’t be neither the criterion of given motion mode self-braking, nor used for reliability estimation for the lack of physical meaning in the self-braking process. The value opposite to the ratio of elementary works of internal resistance forces and driving forces may be considered a correct characteristic of link braking.

4. Self-braking criterion must consider distribution law of friction coefficient as random one. Self-braking reliability evaluation has probability pattern.

### References


In this article, the authors discuss the reliability of self-braking mechanisms in various contexts. They reference several works to support their arguments, including Veits V.L. and Juravleva Ye.Yu’s work on friction forces and self-braking mechanisms, published in 2002. They also cite Turpaev A.I.’s work on self-braking mechanisms from 1976 and Loyniatsky L.G. and Lurie A.I.’s “Course of Theoretical Mechanics” from 1983.

The article concludes with information about the authors, including their academic affiliations and roles within their respective institutions. The authors are from Bauman Moscow State Technical University, and they are listed as: TIMOFEEV Gennadiy Alekseevich, PANJUKHIN Victor Vadimovich, and YAMINSKY Andrey Vladimirovich.