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Многокритериальное управление жизненным циклом процесса металлообработки

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Multi-Criteria Management of the Life Cycle of Metal Cutting Process

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В статье исследован процесс металлической обработки в рамках концепции управления жизненным циклом продукции и технологии, в соответствии с которым, для того, чтобы процесс резания металлов производится эффективно, специалисты вместе задают свои переменные, функциональный ограничения и критерии. В статье рассмотрены разные условия ограничения, относящиеся к температуре, жесткости инструмента и детали. Предлагается методика анализа проблемы многокритериального управления, позволяющие найти решения, когда требования специалистов меняются в процессе согласования. Исходя из результата многокритериального анализа выбираются допустимые варианты производства, которые удовлетворяются требованиям всех экспертов. Данная методика может применяться и в разных инженерных областях и технике.

Ключевые слова: многокритериальное управление, многокритериальная оптимизация, жизненный цикл продукции, процесс резания металлов, токарная обработка, метод визуально-интерактивного анализа.

In this study the authors investigate the metal cutting process based on the concept of product lifecycle and technology management, according to which, for the metal cutting process to take place efficiently, the experts propose their variables, functional constraints, and criteria together. The authors consider various binding conditions related to the temperature, tool stiffness and workpiece. The method of analysis of the multi-objective management problem is proposed, which makes it possible to find solutions when expert requirements change during the reconciliation process. Based on the results of the multi-criteria analysis, acceptable manufacturing options that satisfy the requirements of all the experts are selected. This method can also be used in other engineering and technical fields.

Keywords: multi-criteria management, multi-criteria optimization, product lifecycle, metal cutting process, turning, visual interactive analysis method.

The metal cutting process already exists throughout history of mechanical engineering and manufacturing in the world, yet the process of research and development on this subject is still on-going. The development of this technology which embodies in the depth of physical phenomenon like as friction, heating, chip forming, deformation, destruction, etc., occurring during the turning process. It also makes the modeling exactly of the above nature in mathematical language that enables us to design the metal cutting processes increasingly more optimal.

In terms of optimization, the problem related to turning process has been researched extensively [1–4]. However in nowadays conditions, it is necessary to apply the concept of «Multicriteria management of product life cycle and technology». Because it is the process of working, association, exchanges between experts in the lifecycle of a product and technological process that makes the parameters, the binding conditions, quality criteria being proposed to consider and calculate more precisely than ever before. In other words, quality management of the product life cycle and technology in nowadays are often the multi-criteria problem. The tendency of new period is not «evading», not «simplifying» the important physical nature of the phenomenon, instead of this we will study in depth and expand the multi-dimensional problem one way, from which the product will be created more quality and competitively.

Of course, the expansion of arguments, considering more functional constraints, more interested in quality criterion will make the problem; one side had more options, but on the other side will be more difficult.

In the works of [1–4], the authors consider the problem of optimal three quality criteria for the metal cutting process. However, the strategy that the authors used is compact of three objectives to a single common criteria, then use different algorithms, such as algorithms Cooko [1], Fireflies [1], Hybrid [1, 3], Genetic [2, 4], Swarm, Powell, Brent, CDOS [5–7], etc. to find extreme values of this equivalent objective. Each algorithm aforementioned offers an optimum value with a little difference. In addition, it is noteworthy that, with a minimum value of the equivalent function found, the value of each individual criterion when comparing between different algorithms are dissimilar. That is, there are criteria in a certain algorithm more optimal, but the other criteria, are not equal in other algorithms.

There are two questions that have not been reviewed in detail in the aforementioned researches when applying strategy of the single objective optimization:

– whether really the equivalent objective can substitute for criteria at each separate or not, when the level of importance of each criterion from the perspective of each expert, in one particular moment, in a specific different context of production is different?

– in the real turning process, real production, how the experts to be able to directly analyze, efficiently consider the priority of criteria, to thereby make the appropriate settlement?

The meaning of the optimization algorithms is huge, but in practical conditions, when we need the flexibility to compromise and find a feasible solution of production, the criteria need to be considered separately, repeated many times during the comparative process, next is the process of «concession» to achieve consensus with other criteria. This requires one tool, one method of handling multi-objective problems with high application properties. The following study proposes a method among them Visual Interactive Analysis Method (VIAM).

Problem Statement

Lifecycle process of turning technology in the article shown in Figure 1.

In this simplified cycle, three factors to be considered as customers, technologist engineers and economists.

Variables and their constraints. We will consider three main variables to control the turning process technology:

– v: the cutting speed (m/min), or n: spindle speed(rev/min), $n = 1000v/(\pi D)$, where D is diameter of the work piece (mm);

– f: the feeding rate (mm/rev);

– a: the cutting depth (mm).

In real machining process, the number of variables is much more, for example, the parameters

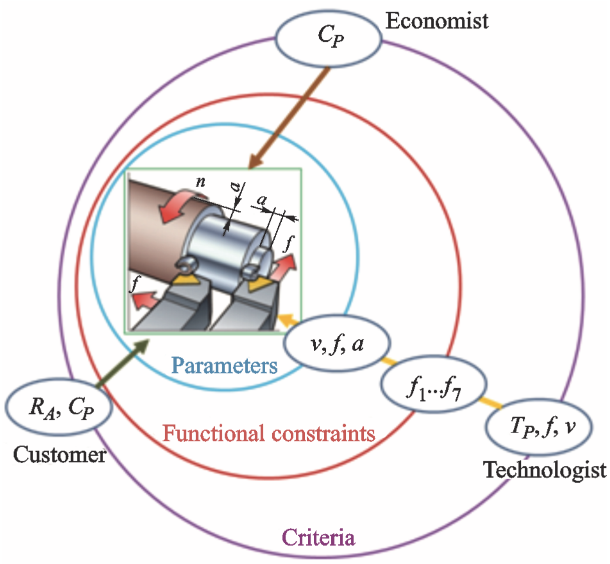

Figure 1. Simplified lifecycle of turning process (The characters in the figure are interpreted below.)

of tool geometry and placement of the tool relative to the work piece, etc. But in this article, we will consider only the above three variables. They are controlled by technological engineers. The other experts are not concerned with these variables.

Their constraints: $v_{\min} \le v \le v_{\max}$; $f_{\min} \le f \le f_{\max}$; $a_{\min} \le a \le a_{\max}$.

Functional constraints. In order for the cutting process is guaranteed in terms of equipment, machinery, tools and surface quality, must satisfy the following constraint conditions.

The conditions on the resistance of turning tool. The ability of the turning tool is determined by its period resistance, which is reflected through cutting speed. This condition under [8–11] has the form

$$f_1 = v - \frac{C_v k_v}{T^{m_v} a^{x_v} f^{y_v}} \le 0,$$

where C_v is the relative strength indicator of tool; k_v is mechanical-physical factor of workpiece; m_v, x_v, y_v are coefficients which characterize machining conditions; T is the period resistance of the tool (min).

Technological and practical requirements relating to the installed power of machines. This condition determines the relationship between the useful power during cutting process, and power of the engine [8–11], it has form

$$f_2 = \frac{C_{P_z} k_{P_z} f^{y_{P_z}} a^{x_{P_z}} (\pi L n)^{n_{P_z}+1}}{6 \cdot 10^{3(n_{P_z}+2)}} - N_L \eta \le 0.$$

Here, the coefficients and factors $y_{P_z}, x_{P_z}, n_{P_z}$ characterizes the level of influence of f, v, a parameters to P_z component of the tool shear force; L is length of workpiece, stuck out from the chuck (mm); N_L is the power of the engine lathe (kW); η is the machine performance.

The conditions of temperature limits, under [9] has form

$$f_3 = C_T a^{xT} f^{yT} v^{zT} - [\Theta] \le 0.$$

Here, the constant C_T and factors x_T, y_T, z_T characterizes the level of influence of v, f, a to the cutting temperature; $[\Theta]$ is temperature limit of turning tool (°C).

The conditions on the resistance of the tool holder. Bending moment generated by P_z component of cutting forces exerted on turning tool not make the normal stresses in the tool exceed the permissible limits. It has form [8–11]:

$$f_4 = \frac{[\sigma] B H^2}{6l} - \frac{C_{P_z} k_{P_z} f^{y_{P_z}} a^{x_{P_z}} n^{n_{P_z}} (\pi D)^{n_{P_z}} k_c}{1000^{n_{P_z}}} \le 0,$$

where $[\sigma]$ is the permissible normal stress of the tool holder (N/mm^2); B and H are width and height of the section of the tool holder (mm); l is the cantilever of the tool holder (mm); k_c is safety factor, secured for the occurrence of complex load.

The conditions on the strength of plate tool:

$$f_5 = C_{P_z} k_{P_z} f^{y_{P_z}} a^{x_{P_z}} - 34 c^{1.35} a^{0.77} \left(\frac{\sin \pi/3}{\sin \varphi} \right)^{0.8} \le 0,$$

where c is thickness of the plate tool (mm); φ is the main angle of the plate in the tool body (rad).

The conditions on the stiffness of tool. To ensure accuracy in turning process and fluctuation limits of the machine, we need to consider the conditions for permissible deflection of the tool:

$$f_6 = 10 C_{P_z} k_{P_z} f^{y_{P_z}} a^{x_{P_z}} v^{n_{P_z}} - \frac{3 E_t I_t [f]_t}{L_{ov}^3} \le 0,$$

where E_t is modulus of elasticity of the tool material (N/mm^2); $I_t = BH^3/12$ is moment of inertia of the tool section (mm^4); $[f]_t$ is permissible deflection of the tool (mm); L_{ov} is the length of the cantilever of plate tool relative to tool body (mm).

The conditions on precision machining. This condition determines the relationship between the calculated value of the cutting speed v, feeding rate f, the cutting depth a and precision machining, which depends on the stiffness of the machine, fixtures, tools and workpiece:

$$f_7 = \frac{10 C_{P_y} k_{P_y} f^{y_{P_y}} a^{x_{P_y}} v^{n_{P_y}} L^3}{\mu E_w I_w} - [f]_w \le 0.$$

Here, the constants C_{P_y}, k_{P_y} and factors $y_{P_y}, x_{P_y}, n_{P_y}$ characterizes the level of influence of f, v, a parameters to P_y component of the tool shear force; μ is coefficient depending on the clamping workpiece method; E_w is modulus of elasticity of workpiece (N/mm^2); I_w is moment of inertia of workpiece section, $I_w = \pi D^4/64$ (mm^4); $[f]_w$ is permissible deflection of workpiece (mm).

All the above functional constraints are managed by technologist engineers. They must be satisfied to ensure cutting machining process takes place safely and accurately.

The quality criteria. Production rate T_p is calculated by the following formulas:

$$T_p = T_s + V_{RM} \frac{1 + T_c / T_L}{MRR} + T_i;$$

$$T_L = \frac{k_T}{v^{\alpha_1} f^{\alpha_2} a^{\alpha_3}}; \quad MRR = 1000vfa,$$

where T_s is the tool set up time (min); V_{RM} is the volume of the removed material (mm³); T_c is the tool change time (min); T_L is the tool life (min); k_T, α_1, α_2, α_3 are constants relevant to a specific combination tool and workpiece [10]; MRR is the material removal rate (mm³/min); T_i is idle time between two consecutive cuts (min).

Operation cost C_p can be expressed as the cost per product, as follows [1–4]

$$C_p = T_p\left(\frac{C_t}{T_L} + C_l + C_o\right),$$

where C_t is the tool cost (€/piece); C_l is the labour cost (€/piece); C_o is the overhead cost (€/piece).

Cutting quality R_a (μm). The criterion for the determination of the surface quality is roughness [1–4]

$$R_a = kv^{k_1} f^{k_2} a^{k_3},$$

where k, k_1, k_2, k_3 are constants relevant to a specific tool–workpiece combination.

Cutting speed v (m/min) and feeding rate f (mm/rev). These are also the important criteria in cutting process.

In terms of technology, the engineer wants the performance T_p has reached the minimum possible value. In addition, the cutting speed v and the feeding rate f should be maximized. Economists are only interested in how to make the production cost reaches a minimum value. Also for the customer, apart from the cost issue, they are most concerned about the cutting quality R_a. So we get a mathematical model of the metal cutting process as follows:

1) the variables: $\mathbf{x} = [x_1 \quad x_2 \quad x_3] = [v \quad f \quad a]$ and their boundary condition:

$$48 \le x_1 \le 120\ (\text{m/min});$$
$$0.01 \le x_2 \le 4.46\ (\text{mm/rev}); \quad 0.1 \le x_3 \le 6\ (\text{mm});$$

2) the functional constraints:

$$f_1(\mathbf{x}) \le 0; \quad f_2(\mathbf{x}) \le 0; \quad f_3(\mathbf{x}) \le 0; \quad f_4(\mathbf{x}) \le 0;$$
$$f_5(\mathbf{x}) \le 0; \quad f_6(\mathbf{x}) \le 0; \quad f_7(\mathbf{x}) \le 0;$$

3) the criteria:

$$\Phi_1(\mathbf{x}) = T_p(\mathbf{x}) \to \min; \quad \Phi_2(\mathbf{x}) = C_P(\mathbf{x}) \to \min;$$
$$\Phi_3(\mathbf{x}) = R_a(\mathbf{x}) \to \min; \quad \Phi_4(\mathbf{x}) = -x_1 \to \min;$$
$$\Phi_5(\mathbf{x}) = -x_2 \to \min;$$

4) the parameters:

$T_s = 0.12$ min; $T_c = 0.26$ min; $T_i = 0.04$ min; $k_T = 1686145.34$; $\alpha_1 = 1.7$; $\alpha_2 = 1.55$; $\alpha_3 = 1.22$; $V_{MR} = 2313.76$ mm³; $C_t = 13.55$ €/piece; $C_l + C_o = 0.39$ €/piece; $k = 1.001$; $k_1 = 0.0088$; $k_2 = 0.3232$; $k_3 = 0.3144$; $D = 100$ mm; $C_v = 420$; $k_v = 0.65$; $T = 30$ min; $m_v = 0.25$; $x_v = 0.15$; $y_v = 0.2$; $C_{P_z} = 300$; $k_{P_z} = 0.77$; $x_{P_z} = 1$; $y_{P_z} = 0.75$; $n_{P_z} = -0.15$; $L = 200$ mm; $N_L = 10$ kW; $\eta = 0.8$; $C_T = 178$; $x_T = 0.08$; $y_T = 0.23$; $z_T = 0.42$; $[\Theta] = 1200$ °C; $C_{P_y} = 243$; $k_{P_z} = 0.75$; $x_{P_y} = 0.9$; $y_{P_z} = 0.6$; $n_{P_y} = -0.3$; $[\sigma] = 15$ N/mm²; $B = 10$ mm; $H = 16$ mm; $l = 1.5 \cdot H$; $k_c = 1.4$; $c = 4.76$ mm; $\varphi = 45°$; $E_t = 2.1 \cdot 10^5$ N/mm²; $[f]_t = 0.1$ mm; $L_{ov} = 1.1 \cdot H$; $E_w = 1.8 \cdot 10^5$ N/mm²; $[f]_w = 0.1$ mm; $\mu = 70$.

We need to find a set of solutions satisfying the constraint conditions and optimize 5 criteria required by experts in the lifecycle of metal cutting technology. In optimization process, experts will have to directly participate in the process of discussion, consideration, compromise and decision.

Method and Algorithm of Solution. The main idea of the VIAM in the article is that: to use the modern methods and algorithms for single-criterion optimization [1–7] to make adjustment tools and search for feasible solutions in the multiple-objectives problem, satisfy the different requirements of each separate expert. The method steps are shown in the below diagram (Figure 2).

Starting from the mathematical model {1}, whereby, need to optimize the $\mathbf{\Phi}$ vector includes the M criteria, subject to functional constraints *constr* which consist of vector $\mathbf{x}$ of variables and vector $\mathbf{f}$ of functions. In step {2}, we use single-objective optimization algorithms are prevalent in the world to find the minimum and maximum value of each separate criterion (subject to the constraints). The obtained values will go to the table {3}.

This is an important interactive table used for the specialists to analyze and conclude in the process of solution finding. Whereby, we know the value domain [minΦ_i; maxΦ_i] of each criterion i. If the final solution is a vector $\mathbf{\Phi}^{\oplus} = \{\Phi_1^{\oplus}, \Phi_2^{\oplus}, \ldots, \Phi_M^{\oplus}\}$ with the value of criteria corresponding to the requirements of experts, it means $\Phi_i^{\oplus} \in [\min\Phi_i; \max\Phi_i]$.

There are two major trends in the search for optimal solution of multi-purpose problem:

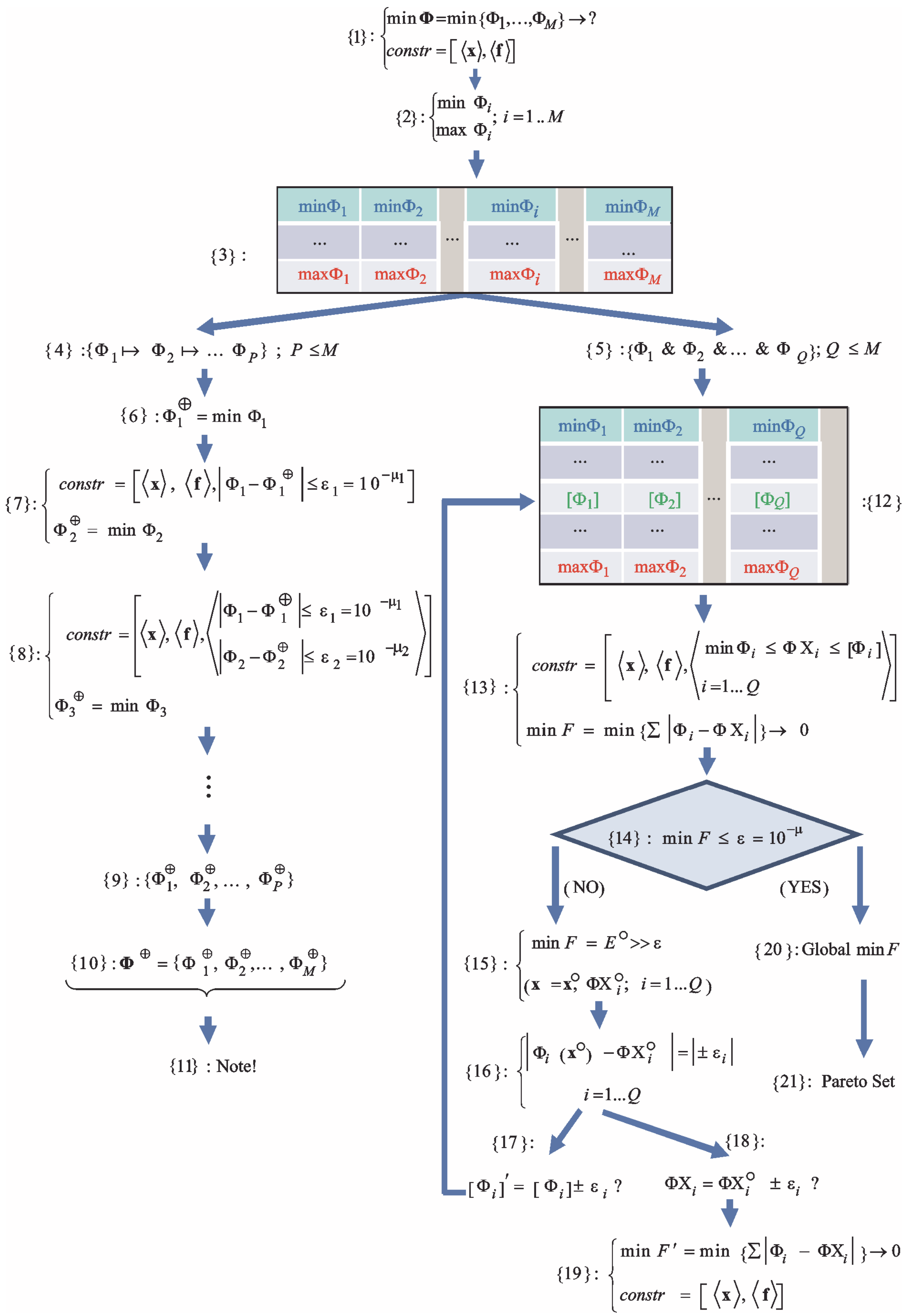

Figure 2. Algorithm of VIAM

– the first trend is predetermined a «hard» priority order for the criteria, e.g. {4}: $\{\Phi_1 \mapsto \Phi_2 \mapsto \ldots \Phi_P\}$; $P \le M$. It means that the first criterion is the most important, it must achieve the best possible value, and then the second criterion will also must to achieve the best its possible value after the concession for the first more important criterion. Same for the next criteria;

– the second trend is the equality of criteria, meaning that there is a group of the most priority criteria with approximately the same importance, e.g. {5}: $\{\Phi_1 \,\&\, \Phi_2 \,\&\ldots\&\, \Phi_Q\}$; $Q \le M$. The concentration for optimization of one criterion in this group might reduce the quality of the other, so in this situation requires the analysis, agreements and concessions between the criteria.

Note, the second trend has a generalized property, it can be transformed into the first trend, where $Q = 1$, or extended for all criteria when $Q = M$. In many cases, although we solve problem based on the second trend, but when it's time to make a final decision, people often accidentally use the first trend. Because in many situations, cannot simultaneously optimize two or more of the criteria, so we must decide to give priority to one of them, albeit reluctantly.

The key idea of the first approach is that, first we optimize the most important criterion; it is the first criterion ({6}). But the best value of the first criterion we had found in advance (= min Φ_1). The problem is that: Maybe among the feasible solutions that allows Φ_1 reaches the value minΦ_1 will have the solution that allows optimize criterion Φ_2, so we can put this value in the form of constraint conditions {7}, from which we can find the optimal possible value of criterion 2. Find this value, we again turn it into the form of constraint conditions from which we can find the optimal possible value of criterion 3 ({8}), etc. And so we come to the criterion P ({9}).

Finally we get the optimal solution according to the order of the desired priority ({10}). Note {11}: There can be many vector of parameter that allows achieving this solution. Although a certain criterion can achieve the global optimization value, but is it necessarily should have that value, when, if we just reduce a little the threshold of this criterion, it was able get much more optimal solutions for other criteria? The experts could not have known this, but only the adjustment tools to widen the new constraint conditions helps us to accurately assess. Thereby, it is very likely we can gain good solutions for other criteria.

The second trend, which is a group of Q criteria with the same level of importance. Now, each expert will must define a «threshold» value $[\Phi_i]$ for his i-th criterion in the interactive table {12}. So, the set of feasible solutions will be the vectors $\mathbf{\Phi X} = \{\Phi X_1, \Phi X_2, \ldots, \Phi X_M\}$ with criteria value satisfy the condition $\Phi X_i \in [\min \Phi_i; [\Phi_i]]$. The main problem is that we need to find the set of these $\mathbf{\Phi X}$ vectors. The way to solve this problem is that we will change the above requirements become the additional constraints {13}, whereby, optimization problem becomes to find not only the N parameters x_j ($j = 1...N$), but also to find more the Q values ΦX_i ($i = 1\ldots Q$) when we optimize the function $\min F = \min\{\sum|\Phi_i - \Phi X_i|\} \to 0$. {14} if the optimal value of this F function seeks to 0, that is likely exist many other solution vectors. We will use one–criterion optimization algorithms to find all of these solutions; they are set of the Pareto solutions ({20} and {21}). But mostly not so easily we fall into such favorable circumstances. Because maybe these threshold values $[\Phi_i]$ are fairly easy or they randomly suitable for appearance of the solutions. But in most cases, we will fall into the situation {15}, when the minimum value of the function cannot be carried to 0. If we believe that the one-criterion optimization algorithms today to find min$F$ is strong enough, then the event that min$F$ cannot seek to value 0 means: in the limit of threshold values that experts specified, we cannot find any feasible solution. This is the most difficult and also the most interesting situations. There are three questions should be put:

– really the thresholds of the criteria $[\Phi_i]$ that experts set out in the table {12} are correct and reasonable?

– if indeed we cannot find any feasible solutions, so what we have to do?

– if we have to change the value of the criteria threshold, for what criteria should we change? And how we will change (increase or decrease?) and how much changing?

Experts set the threshold values almost based on the production experience with their own subjective perspective, so, before a problem solution, they themselves also cannot know what the threshold value is correct. Thus, the only way to solve the problem is to change the threshold value. But if so, then the 3rd question must be answered. To answer this question, we move to step {16}, we must calculate the difference between the values of the criteria function, calculated by the parameters compared with its target value. But the feature of the method is that, results is not in the form of last value. It should be in the form $|\pm\varepsilon_i|$ (i = 1...Q). Plus or minus sign will tell us that need to change the threshold values in increasing or decreasing order. If the plus, we need to increase, on the contrary we need to decrease the value. Value ε_i tells us how much will have to change the magnitude ΦX_i. If the difference of a certain criterion is approximate to 0, it means that the threshold value of criteria in relation to the value of the other criteria is consistent and does not need to change.

Here we have two ways to solve the problem: The first way is to change the threshold value as mentioned above {17}, the new threshold is brought back to the table {12} to continue the process of calculating. 2nd direction, we will find specific solutions directly with the new threshold value ({18} and {19}). According to the first solving direction, when the new threshold value is appropriate, we will move to the steps {20} and {21} to find a set of Pareto solutions, it is the set of solutions that cannot be more optimized for all criteria at the same time. It should be noted that, in step {17}and {18}, changing the threshold value is an important step; it will determine the existence or non-existence of a feasible solution of the problem, therefore, necessary to have the involvement and comments of experts in the field of solving problem.

Let's consider an example of the multi-objective management of the metal cutting process that its mathematical model was set up at the beginning of the article.

Results and Discussion. 1. Step 1: Determine the minimum and maximum value of each separate criterion to tabulate {3}.

Using the modern methods and algorithms for single-criterion optimization [1–7, 12, 13], we find these extremal values and make the Table 1.

Looking at Table 1, we see the value domain of the two criteria Φ_1 and Φ_2 is narrow, which means superior capabilities of the solution compared with each other based on the two criteria are not significant.

2. Trend 1: Assuming that the experts after discussion agreed that the importance order of these criteria is to be achieved as $\{\Phi_5 \mapsto \Phi_4 \mapsto \Phi_3\}$, the first two criteria Φ_1 and Φ_2 are freedom.

First, we had minΦ_5, so initially, we set $\Phi_5^{\oplus} = \min \Phi_5 = -1.57756339337772$. We add to the constraints *constr* the condition $|\Phi_5 - \Phi_5^{\oplus}| \le \varepsilon_5 = 10^{-6}$ to find min Φ_4. We obtained results minΦ_4 = –48.0005417699204457 when x_1 = 48.0005417701829, x_2 = 1.57756239828247, x_3 = 1.04694948597870.

But the technology engineers found that if we reduce the target of criterion Φ_5 a little, the metal cutting process will not be affected significantly, so, we set $\Phi_5^{\oplus} = -1.54$. Find the extreme values for Φ_4 criterion but the constraints will also change, we obtained minΦ_4 = –71.8521289608874128 with x_1 = 71.8521289611547, x_2 = 1.53999900729847, x_3 = 1.13254204048779. It was found that only reduced the criterion 1 about 2.38% we can optimize criterion 4 up 49.7%.

Next, to try to see whether we can optimize criterion 4 anymore, we set $\Phi_5^{\oplus} = -1.51$. We obtained results minΦ_4 = –73.0316940583404630 when x_1 = 73.0316940586067, x_2 = 1.50999900741839, x_3 = 1.15218688270014. At this point, we see the criterion 4 is optimized up slightly (1.64%), while the main criteria to be reduced even more (1.95%). Therefore, the experts decide: $\Phi_5^{\oplus} = -1.54$, $\Phi_4^{\oplus} = -71.852$.

We add to the constraints *constr* the conditions $|\Phi_5 - \Phi_5^{\oplus}| \le \varepsilon_5 = 10^{-6}$ and $|\Phi_4 - \Phi_4^{\oplus}| \le \varepsilon_4 = 10^{-6}$ to find minΦ_3. We obtained results minΦ_3 = = 1.24271641374314 with x_1 = 71.8521279611550, x_2 = 1.53999900729847, x_3 = 1.13254203812347.

Table 1

The interactive table

minΦ_1 = = **0.17017491334135906**	minΦ_2 = = **0.07165667530465312**	minΦ_3 = = **0.9933997276142956**	minΦ_4 = = **–120**	minΦ_5 = = **–1.57756339337772**
…	…	…	…	…
[Φ_1]	**[Φ_2]**	**[Φ_3]**	**[Φ_4]**	**[Φ_5]**
…	…	…	…	…
maxΦ_1 = = **0.212231347853060**	maxΦ_2 = = **0.0833716578409044**	maxΦ_3 = = **1.64169482078076623**	maxΦ_4 = = **–48**	maxΦ_5 = = **–0.1538219056520352**
Production rate (min)	Operation cost (€/piece)	Cutting quality (μm)	Cutting speed (m/min)	Feeding rate (mm/rev)

So, in the order of priority $\{\Phi_5 \mapsto \Phi_4 \mapsto \Phi_3\}$, we obtain the following solution:

$$\mathbf{\Phi}^{\oplus} = \{\Phi_1^{\oplus} = 0.1784723362; \Phi_2^{\oplus} = 0.07427258407; \Phi_3^{\oplus} = 1.242716415; \Phi_4^{\oplus} = -71.85212796; \Phi_5^{\oplus} = -1.539999\}. \quad (1)$$

The variables:

$$\mathbf{x}^{\oplus} = \{x_1 = 71.8521279611550; x_2 = 1.53999900729847; x_3 = 1.13254203812347\}.$$

3. Trend 2: Assume that the experts agreed that the following group of the criteria is the most important $\{\Phi_3 \,\&\, \Phi_4 \,\&\, \Phi_5\}$.. These three criteria are equally important. According to production experiences, the experts made the permissible threshold value of these criteria in the table {12}: $[\Phi_3] = 1.2$; $[\Phi_4] = -85$; $[\Phi_5] = -1.4$. We add to the constraints *constr* the three conditions $\min \Phi_3 \le \Phi X_3 \le [\Phi_3]$, $\min\Phi_4 \le \Phi X_4 \le [\Phi_4]$, $\min\Phi_5 \le \Phi X_5 \le [\Phi_5]$ to find extreme values for this function

$$\min F = \min\{|\Phi_3 - \Phi X_3| + |\Phi_4 - \Phi X_4| + |\Phi_5 - \Phi X_5|\} \to 0.$$

We obtained results

$$\min F = E^{\circ} = 0.383977641260542 >> 10^{-6}$$

where:

$$\mathbf{x}^{\circ} = \{x_1^{\circ} = 85.0000001757532; x_2^{\circ} = 1.02681910527132; x_3^{\circ} = 1.57405934675711\};$$
$$\Phi X_3^{\circ} = 1.2; \quad \Phi X_4^{\circ} = -85.0000001808302; \quad \Phi X_5^{\circ} = -1.4.$$

It means that there did not exist a feasible solution at the domain of the threshold value which experts set. So need to start the process of analyzing and compromise. First of all, to answer the three mentioned above questions, we need calculate the difference between the values of the criteria function, calculated by the parameters compared with its target value:

$$|\Phi_3(\mathbf{x}^{\circ}) - \Phi X_3^{\circ}| = |+0.0107967325372993| = |+\varepsilon_3| > 10^{-6};$$
$$|\Phi_4(\mathbf{x}^{\circ}) - \Phi X_4^{\circ}| = |+5.33708622246534 \cdot 10^{-9}| = |+\varepsilon_4| < 10^{-6};$$
$$|\Phi_5(\mathbf{x}^{\circ}) - \Phi X_5^{\circ}| = |+0.373180903386157| = |+\varepsilon_5| > 10^{-6}.$$

We understand that, with the threshold value of criteria 3 and 5 which experts set we cannot obtain a feasible solution. So, the only way is the experts have to accept an adjustment in these two criteria, otherwise, they must make the totally other threshold values and solve the problem again because this is a situation of force majeure. Assume that they approve the adjustment because the difference is not too much, we will try to follow the direction {18}, we search directly the solution at the adjusted threshold value, i.e. with the initial constraints of the problem {1}, we set $\Phi X_3 = \Phi X_3^{\circ} + \varepsilon_3$; $\Phi X_4 = \Phi X_4^{\circ} + \varepsilon_4$; $\Phi X_5 = \Phi X_5^{\circ} + \varepsilon_5$ to find the extreme of the function

$$\min F' = \min\{|\Phi_3 - \Phi X_3| + |\Phi_4 - \Phi X_4| + |\Phi_5 - \Phi X_5|\} \to 0.$$

We obtained results:

$$\mathbf{\Phi}^{\oplus} = \{\Phi_1^{\oplus} = 0.176850605617297; \Phi_2^{\oplus} = 0.0738790560641888; \Phi_3^{\oplus} = 1.210796733; \Phi_4^{\oplus} = -85; \Phi_5^{\oplus} = -1.026819097\}. \quad (2)$$

The variables:

$$\mathbf{x}^{\oplus} = \{x_1 = 85; x_2 = 1.02681910565741; x_3 = 1.57405934815806\}.$$

Comparing with the solution (1), we see only the criterion 5 in (2) is worse than in (1), while all the remaining criteria are better. This is logical because in the solution at (1) we set the criterion 5 as the most important. In solving (2) we believed that the importance of the criteria is the same. Here, the criteria 1 and 2 are optimized simultaneously with 3 remaining criteria, although experts did not give a threshold value for these two criteria.

4. Trend 2: Let's see another situation to test ability to work of the VIAM. Suppose that the experts agreed the following group of criteria is the most significant $\{\Phi_1 \,\&\, \Phi_2 \,\&\, \Phi_3\}$. These three criteria are equally important. According to their own production experience, experts give permission threshold value of these three criteria in the table {12}: $[\Phi_1] = 0.171$; $[\Phi_2] = 0.073$; $[\Phi_3] = 1.1$. We add to the constraints *constr* the three conditions $\min\Phi_1 \le \Phi X_1 \le [\Phi_1]$, $\min\Phi_2 \le \Phi X_2 \le [\Phi_2]$, $\min\Phi_3 \le \Phi X_3 \le [\Phi_3]$ to find extreme values for the function

$$\min F = \min\{|\Phi_1 - \Phi X_1| + |\Phi_2 - \Phi X_2| + |\Phi_3 - \Phi X_3|\} \to 0.$$

We obtained results

$$\min F = E^{\circ} = 0.108447846899959$$

with

$$\mathbf{x}^{\circ} = \{x_1^{\circ} = 84.1145659326976; x_2^{\circ} = 0.944343931136707; x_3^{\circ} = 1.67344619247632\};$$
$$\Phi X_1^{\circ} = 0.171; \quad \Phi X_2^{\circ} = 0.073; \quad \Phi X_3^{\circ} = 1.1.$$

Table 2

The set of global solutions

No.	$\Phi_1^{\oplus} \to \min$	$\Phi_2^{\oplus} \to \min$	$\Phi_3^{\oplus} \to \min$	$\Phi_4^{\oplus} \to \min$	$\Phi_5^{\oplus} \to \min$
1	0.172650965	0.073005519	1.201261252	–113.9168125	–0.387479442
2	0.172789574	0.073251972	1.201236981	–113.1405204	–0.449652358
3	0.172958920	0.073119402	1.193249525	–113.8579187	–0.420150017
4	0.172979769	0.073291636	1.196050245	–113.1328238	–0.464956059
5	0.173219771	0.072510994	1.197785006	–109.2873286	–0.318547229
6	0.173290776	0.073109935	1.182397663	–114.3205247	–0.422940935
7	0.173301383	0.073304491	1.190783431	–112.0510616	–0.483831386
8	0.173739229	0.072395940	1.153546657	–118.181637	–0.265804087
9	0.173744851	0.073360959	1.194764383	–107.4997202	–0.540895498
10	0.173763523	0.073346618	1.190558805	–108.5299449	–0.529711341
11	0.173766994	0.073077816	1.166838480	–115.102587	–0.418792721
12	0.173827903	0.072563122	1.154398884	–117.5118445	–0.298460533
13	0.173905728	0.072128565	1.197530753	–103.3056765	–0.269050457
14	0.173943959	0.073248008	1.164705017	–114.6369197	–0.466182671
15	0.173982740	0.072238872	1.153748632	–115.7343727	–0.245839466
16	0.174006644	0.073272836	1.164450512	–114.2582402	–0.475846421
17	0.174062635	0.073418831	1.201091535	–103.4937397	–0.596752497
18	0.174102443	0.072713470	1.149102810	–117.2378717	–0.329404112
19	0.174167176	0.072202262	1.139395742	–118.7255689	–0.229729519
20	0.174201229	0.072205132	1.134680653	–120.0000000	–0.226393488
21	0.174222705	0.073433605	1.200170313	–102.6263622	–0.611930628
22	0.174245196	0.072220666	1.133673270	–120.0000000	–0.228558723
23	0.174304607	0.072279458	1.133214009	–119.7913333	–0.238380670
24	0.174331257	0.072260941	1.131955683	–119.9424898	–0.234633558
25	0.174386227	0.073080426	1.148108491	–115.991716	–0.419321650
26	0.174403862	0.073216260	1.150287121	–115.4211834	–0.457645985
27	0.174415253	0.073264254	1.150890247	–115.2310437	–0.471765207
28	0.174424002	0.073425238	1.191429618	–103.5617483	–0.603987755
29	0.174483087	0.072244401	1.127858716	–120.0000000	–0.230318203
30	0.174590641	0.073325344	1.158762105	–111.5666779	–0.513334068
31	0.174666966	0.072822824	1.134419477	–117.6384074	–0.349706998
32	0.174693630	0.072197243	1.122302168	–120.0000000	–0.220168495
33	0.174732590	0.072674267	1.129234088	–118.4917709	–0.313459941
34	0.174755175	0.072937859	1.134262417	–117.2292609	–0.377770660
35	0.174760659	0.073026317	1.135950629	–116.8200924	–0.401341690
36	0.174800714	0.072322102	1.120596230	–120.0000000	–0.240001130
37	0.174822740	0.072303377	1.119924657	–120.0000000	–0.236455100
38	0.174825837	0.072604530	1.124903213	–119.0193054	–0.296224607
39	0.174856049	0.072294202	1.119048950	–120.0000000	–0.234428812
40	0.174915013	0.073525543	1.200990040	–97.88179927	–0.698733994
41	0.174945206	0.072599624	1.121257051	–119.2641133	–0.293026341
42	0.174996670	0.073534917	1.200408591	–97.51939798	–0.706689427
43	0.175012536	0.073416888	1.167300681	–106.1595669	–0.583322755
44	0.175029442	0.073504689	1.190779210	–99.75431292	–0.672093657
45	0.175108136	0.072825836	1.121551863	–118.3816588	–0.343433065
46	0.175385072	0.072427214	1.107441318	–120.0000000	–0.250316140
47	0.175462975	0.072635899	1.107137102	–120.0000000	–0.290401734
48	0.175501822	0.072314827	1.103880694	–120.0000000	–0.227578580
49	0.175525894	0.073157837	1.148988968	–107.4195303	–0.485335209
50	0.175644717	0.073635357	1.201087311	–93.50832631	–0.795018902
51	0.175670531	0.073640108	1.201259885	–93.32052916	–0.799352754
52	0.175887670	0.072389548	1.095612397	–120.0000000	–0.233187704

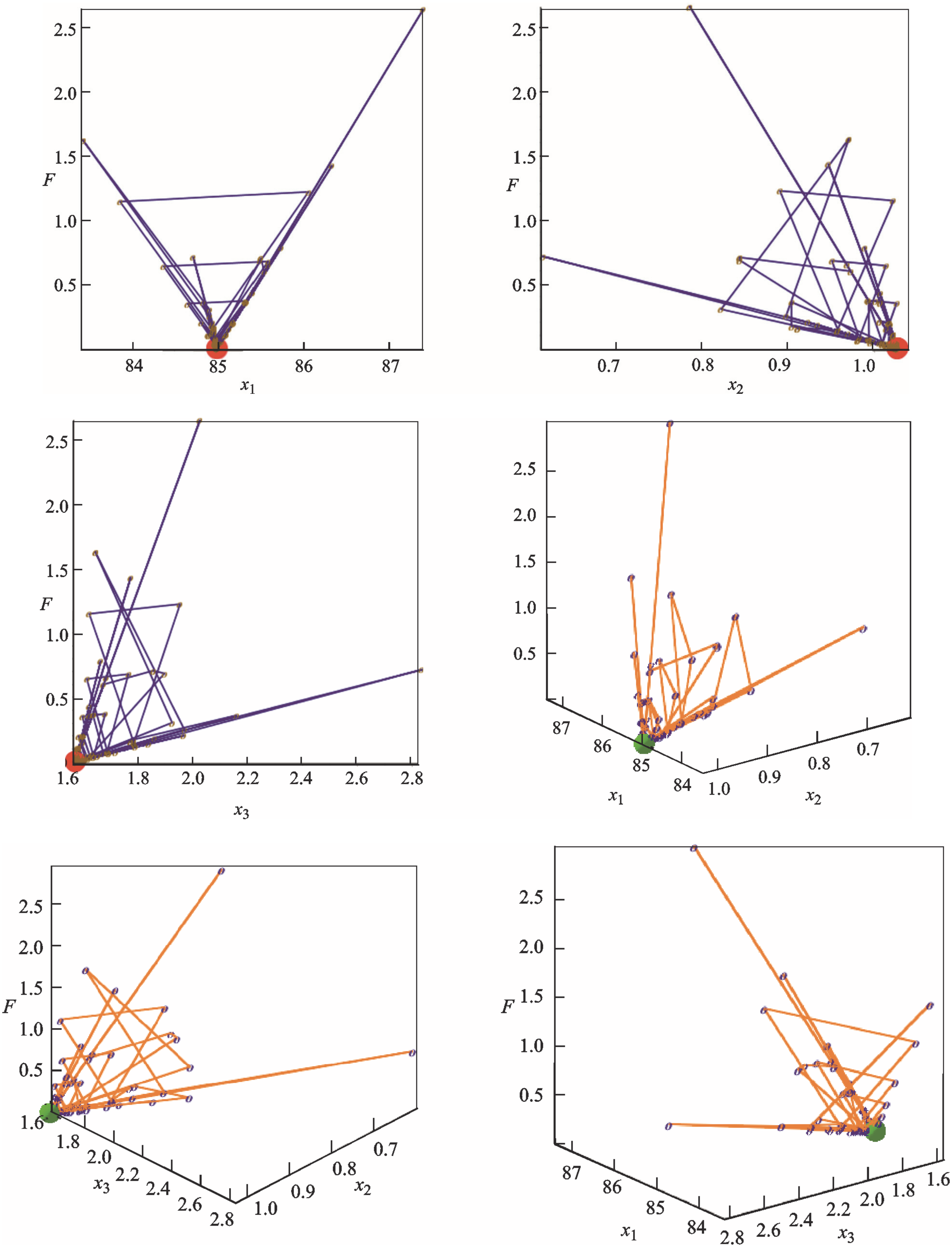

Figure 3. The illustration of the search path in plane and space

The difference between the values of the criteria function, calculated by the parameters compared with its target value:

$$|\Phi_1(\mathbf{x}^{\circ}) - \Phi X_1^{\circ}| = |+0.00641486384| = |+\varepsilon_1'|;$$

$$|\Phi_2(\mathbf{x}^{\circ}) - \Phi X_2^{\circ}| = |+0.00076871859| = |+\varepsilon_2'|;$$

$$|\Phi_3(\mathbf{x}^{\circ}) - \Phi X_3^{\circ}| = |+0.101264265| = |+\varepsilon_3'|.$$

Looking at the value of these differences, experts decided they are acceptable. So, with the original constraints and the new threshold value: $[\Phi_1]' = [\Phi_1] + \varepsilon_1'$; $[\Phi_2]' = [\Phi_2] + \varepsilon_2'$; $[\Phi_3]' = [\Phi_3] + \varepsilon_3'$ to find the global extreme values for this function

$$\min F' = \\ = \min\{|\Phi_1 - \Phi X_1| + |\Phi_2 - \Phi X_2| + |\Phi_3 - \Phi X_3|\} \to 0.$$

We obtained the following set of global solutions (Table 2).

The above solutions are the Pareto set; we cannot simultaneously optimize all of these criteria at the same time. In other words, in all the above solutions, there is at least one group of criteria is better than all other solutions and also have at least one group of criteria is worse than all other solutions. The experts will select one of these 52 solutions the most appropriate for them, they are

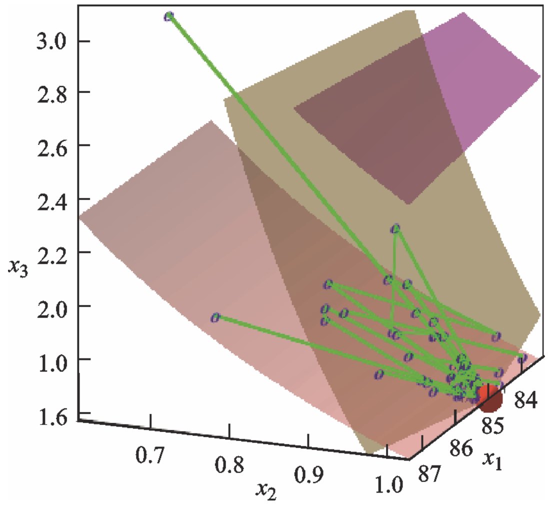

Figure 4. The search path in the space of variables and functional constraints (The red sphere represents the optimal value of the function.)

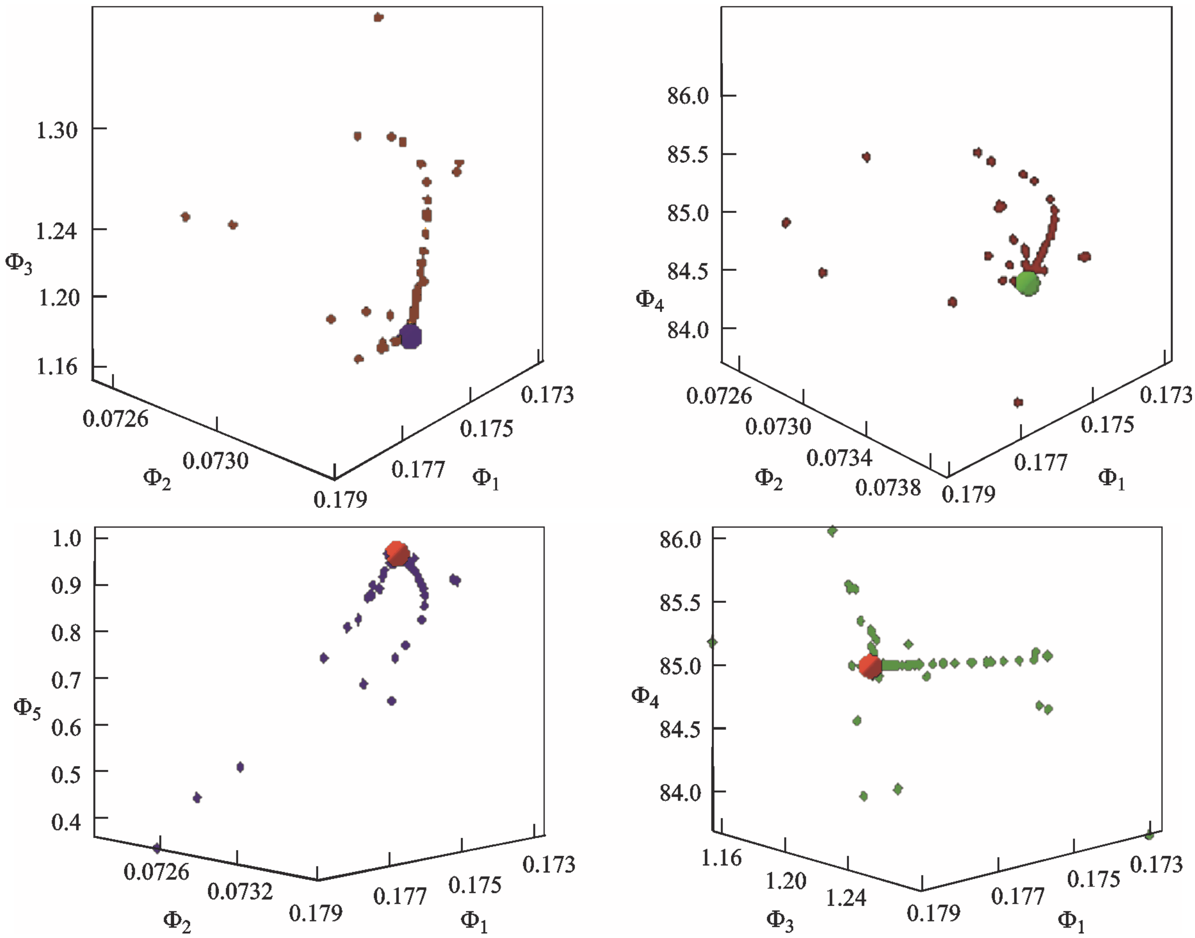

Figure 5. The location of solution in the 5-dimensional criteria space

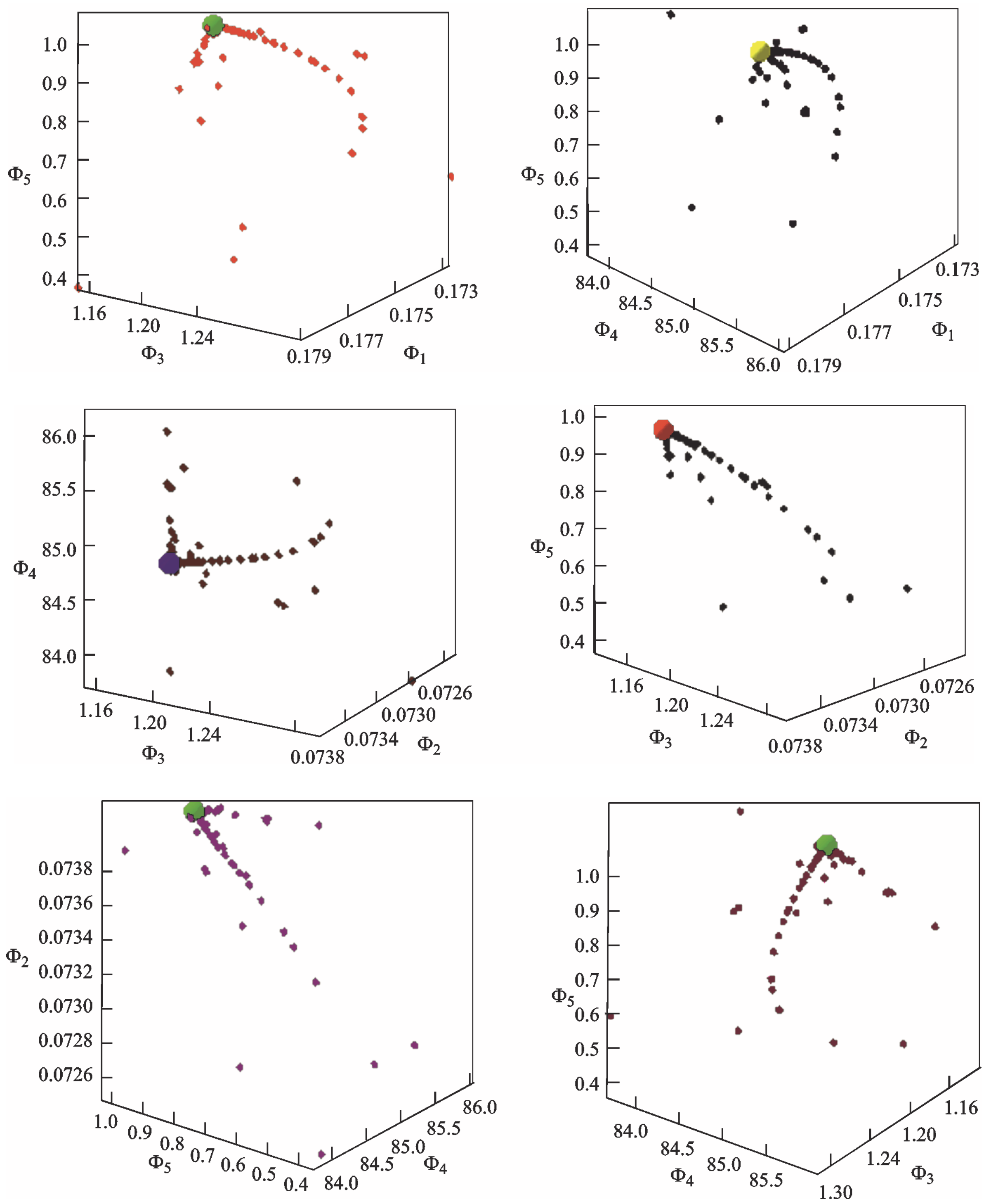

Figure 5. The location of solution in the 5-dimensional criteria space

also the optimal solutions with the minor difference. But comparing these 52 solutions with the (1) and (2), we see though most the criteria 1, 2, 3, 4 are better, but the criterion 5 is worse too much. This is reasonable because at 52 solutions in the table the experts did not focused their attention on the criterion 5, they only care about the first 3 criteria. The interesting thing is that when we optimized the first 3 criteria, we see that the criterion 4 is also optimized. It means that: If we increase the cutting speed of the workpiece, we must decrease the feeding rate of the tool, and then the three criteria as the production rate, the operation cost and cutting quality will be also op-

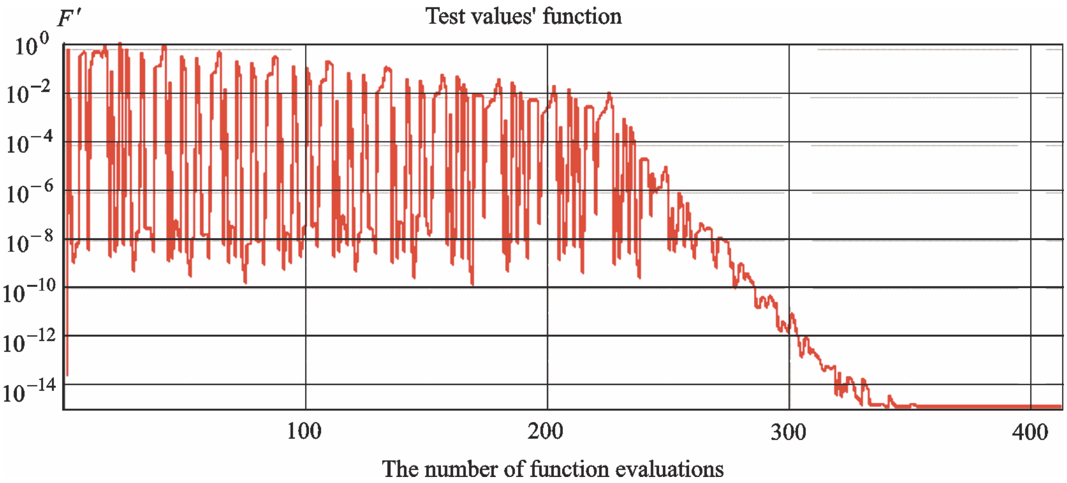

Figure 6. The process of approaching to the target values of the function F'

timized, but each criterion cannot achieve the optimal value as each separately in this overall relation.

Let's see the illustration of the path to the solution (2) when optimize the function

$$\min F' =$$

$$= \min\{|\Phi_3 - \Phi X_3| + |\Phi_4 - \Phi X_4| + |\Phi_5 - \Phi X_5|\} \to 0.$$

Use the published methods and algorithms for single-criterion optimization [1–7], this search path undergone 413 times calculating the function F' to compare the value.

In Figures 3 was shown the illustration of the search path in plane and space. The numbers indicate the step number of the function calculation. The red circulars and the green spheres represent the optimal value of the function.

The search path in the space of variables and functional constraints (the surfaces) shown in Figure 4.

In Figure 4 we only see the surface of the three constraint functions, the other functions did not appear because they are outside of the search space, they are always satisfied in the search path shown in this figure. We see that when searching, the test points although move but cannot exceed the surface of the functional constraints.

In the 5-dimensional criteria space let us consider each of the three criteria to know the location of solution in 3D space (Figure 5). The big spheres represent the optimal value of the function.

The graph shows the values of function F' approaching the target after 413 optimal steps shown in Figure 6.

Conclusions

For a solution to the problem of multi–objective optimization, which is used in quality management of the product lifecycle and technology, the answer is never the unique. Because the solution is a set of multi–criteria values, but each criterion has different importance from the perspective of different specialists, in each different circumstance. Therefore, the evaluation of the certain optimal solution is better than the others, based on the value of an equivalent function for all criteria, is not comprehensive.

It should be noted in the article is that: if the requirements of the experts on the importance of criteria will change in a context of other production conditions, or threshold values of criteria are adjusted, with the proposed above VIAM, it is easily to solve and analyze the problem several times depending on the needs of experts. And of course, the obtained solution will differ from the previous, but we cannot say what the solution is superior, because they only satisfy the specific requirements of a group of experts at a certain production time. It is important that the method proposed in the article could allow the experts accurately estimate and make rational decisions for all the time with different requirements.

We've found 54 solutions for the situation with the different criteria requirements in the process of searching for optimal solutions of the metal cutting process. All of them are the Pareto solution. The problem also can be expanded with the parameters and constraints, as well as other criteria. The VIAM allows solve the multi–objective problems with the arbitrary complexity.

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